Mechanised Semantics of BSP Routines with Subgroup Synchronisation

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University of Paris-East

Outline

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Bridging Model: Bulk Synchronous Parallelism (BSP)

The BSP computer

Defined by:

- **p** pairs CPU/memory
- Communication network (**g**)
- Synchronisation unit (**L**)
- Super-steps execution

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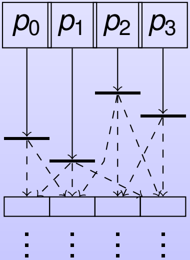

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barrier ($\oplus$**L**)

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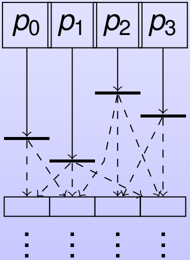

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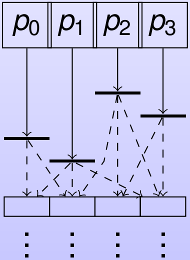

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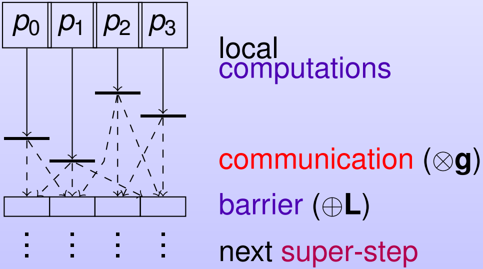
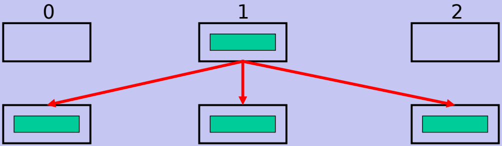


Examples: broadcasting a values

Direct broadcast (one super-step)



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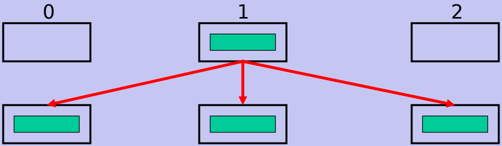

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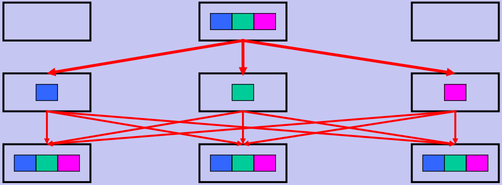
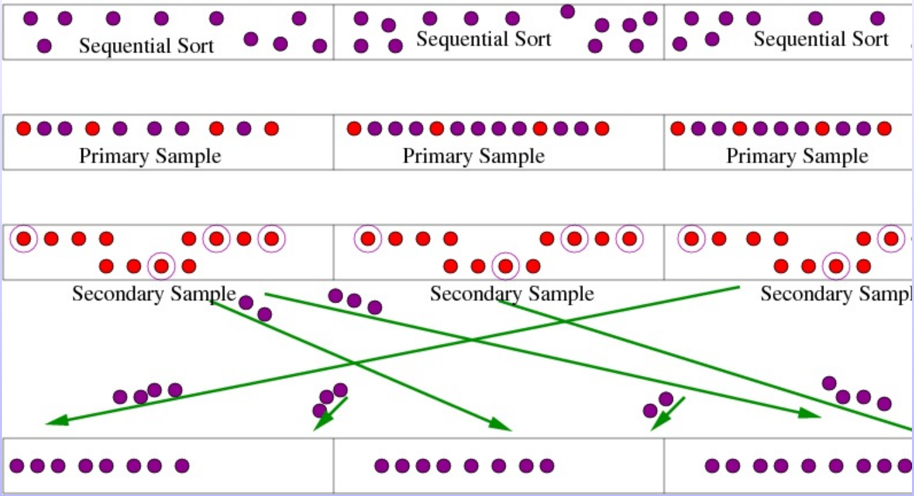
Broadcast with two super-steps



Cost $\equiv 2 \times \mathbf{g} \times n \; + \; 2 \times \mathbf{L}$

Parallel Sorting by Regular Sampling (PSRS)



BSP Imperative Programming

Languages and libraries

1. Dedicated Languages: NestStep, BSP++, BSP-Python, . . .
2. BSPLib for C and Java
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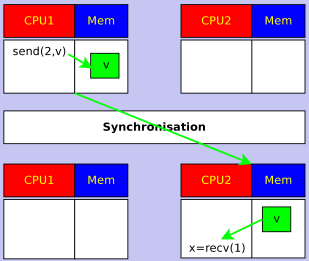

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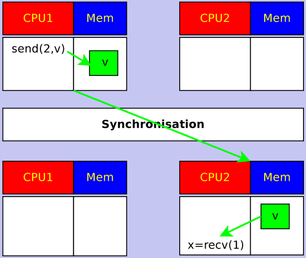

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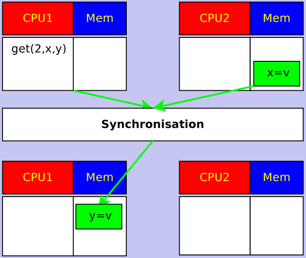

Examples of C primitives

BSMP and DRMA

- Typical BSMP routines:
  - `bsp_send(grp,dest,buffer,size)`
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  - `msg* bsp_findmsg(grp,proc_id,index)`
- Typical DRMA routines:
  - `bsp_push_reg(grp,ident,size)`
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- `bsp_sync(grp)` (barrier)

MPI collective operations

`MPI_Scatter(sendbuf,sendcount,sendtype,recvbuf,recvcount,recvtype,root,comm)`

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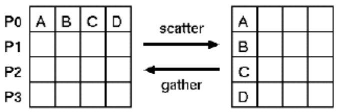

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Common Bugs in BSP Imperative Programs

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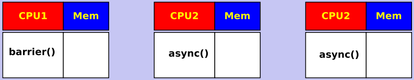

Data race, non-determinism

Out-of-bound errors

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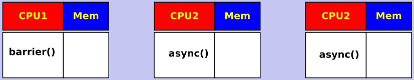

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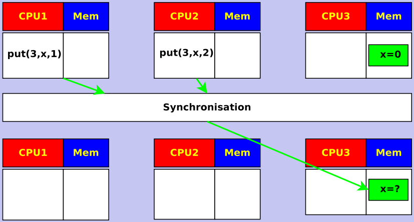

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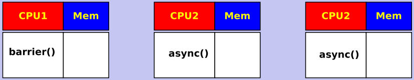

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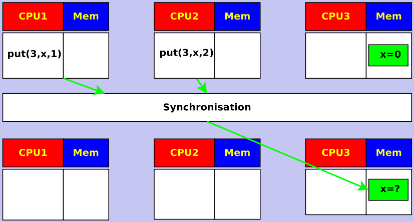

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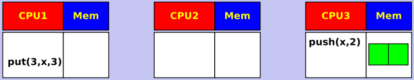


Subgroup Synchronisation

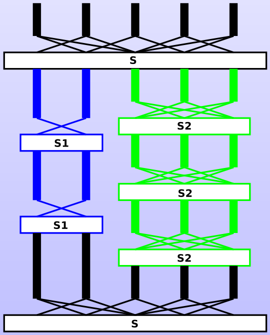
Allows the synchronisation of a subset of processes

Advantages

- Take advantage of hybrid models (better performance)
- Close to MPI's collective operations

Drawbacks

- More complex programs
- BSP cost model is lost



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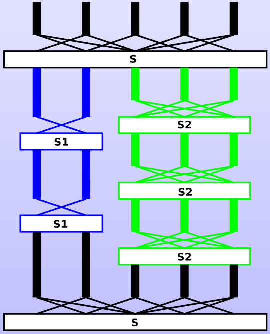

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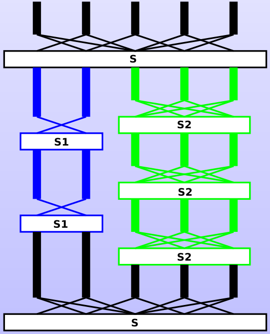

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Using the PUB

- `BSP_WORLD:t_bsp` ⇒ all the processors
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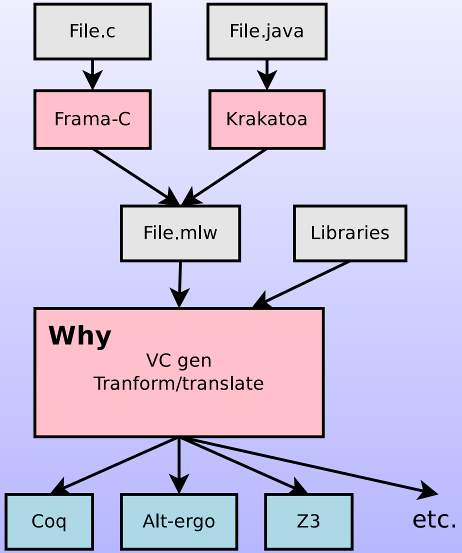
File.c
File.java
Frama-C
Krakatoa
File.mlw
Libraries
Why
VC gen
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Coq
Alt-ergo
Z3
etc.

The WhyML Language for Deductive Verification

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let sqrt (n:int) =
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  let sum = ref 1 in
  while !sum ≤ n do
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  - Additional instructions for parallel operations
  - Additional notations in assertions about parallelism
- Automatic transformation to Why code (sequentialisation)

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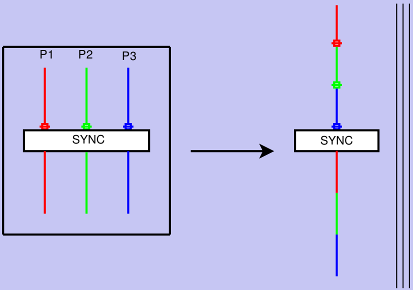

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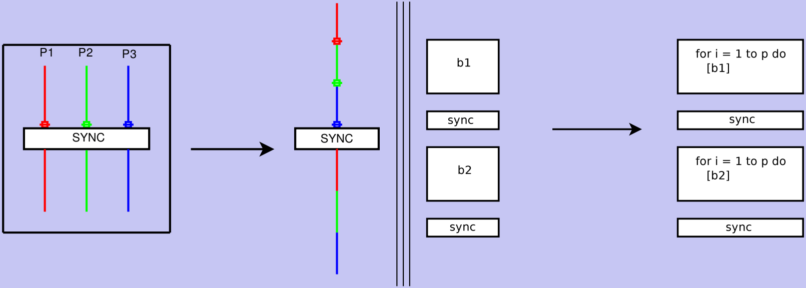

The BSP-Why tool (2)

Language definition

BSPWhy	::=	*Why*	
	|	**sync**	synchronisation and parameters
	|	**push**(x)	Register x for global access
	|	**put**(e, x, y)	Distant writing
	|	**send**(x, e)	Message passing

Logic extensions

- x, to represent the value of x on the current processor
- $x < i >$, to represent the value of x on the processor i
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- Big-step (natural) semantics are defined as a reference
- Co-inductive semantics for infinite programs

Small-step semantics

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- All semantics defined in Coq
- Inductive and CoInductive definitions

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- Exclusive between finite and infinite rules
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Big-Step Semantics: Examples of Local Rules

$$\frac{}{s, \textbf{pid} \Downarrow^i s, i} \qquad \frac{}{s, \textbf{nprocs} \Downarrow^i s, \textbf{p}}$$

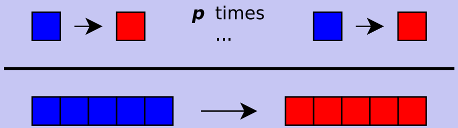
$$\frac{s, e_1 \Downarrow^i s', v \qquad s'[x \leftarrow v], e_2 \Downarrow^i s'', o}{s, \textbf{let } x = e_1 \textbf{ in } e_2 \Downarrow^i s'', o}$$

$$\frac{s, e_1 \Downarrow^i s', \textbf{SYNC}(C, e')}{s, \textbf{let } x = e_1 \textbf{ in } e_2 \Downarrow^i s', \textbf{SYNC}(C, \textbf{let } x = e' \textbf{ in } e_2)}$$

$$\frac{s, e \Downarrow^i s', \textbf{SYNC}(C, e')}{s, x := e \Downarrow^i s', \textbf{SYNC}(C, x := e')}$$

Big-Step Semantics (without subgroup)

Execution without synchronisation



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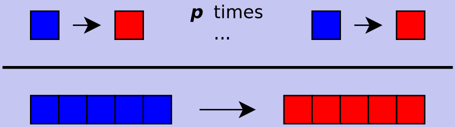

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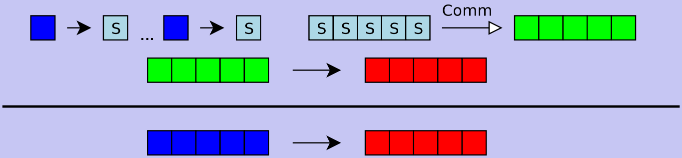

Diverging Big-Step Semantics (without subgroup)

$$\frac{\exists i \quad s_i, e_i \Downarrow^i_\infty}{\langle (s_0, e_0), \ldots, (s_{\mathbf{p}-1}, e_{\mathbf{p}-1}) \rangle \Downarrow_\infty}$$

$$\frac{\forall i \quad s_i, e_i \Downarrow^i s'_i, \mathbf{SYNC}(e'_i) \qquad \mathbf{AllComm}\{\langle (s'_0, e'_0), \ldots, (s'_{\mathbf{p}-1}, e'_{\mathbf{p}-1}) \rangle\} \Downarrow_\infty}{\langle (s_0, e_0), \ldots, (s_{\mathbf{p}-1}, e_{\mathbf{p}-1}) \rangle \Downarrow_\infty}$$

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Small-Step Semantics (without subgroup)

Naive solutions

- $\langle \ldots (s_i, \textbf{bsp\_sync}; e_i) \ldots \rangle \rightharpoonup \langle \ldots (s'_i, e_i) \ldots \rangle$
  $\Rightarrow$ Impossible to evaluate: **if** b **then bsp_sync else** e
- $s, \textbf{bsp\_sync} \overset{i}{\rightharpoonup} s, \mathit{Wait}(\textbf{skip})$
  $\langle \ldots (s_i, \mathit{Wait}(e_i)) \ldots \rangle \rightharpoonup \langle \ldots (s_i, e_i) \ldots \rangle$(Tesson, Loulergue)
  $\Rightarrow$ Impossible to evaluate: $(e_1; \textbf{bsp\_sync}); e_2$
- Congruence "$(e_1; \textbf{bsp\_sync}); e_2 \equiv e_1; (\textbf{bsp\_sync}; e_2)$" (Fortin, Gava)
- Use of contexts $\langle \ldots \Delta[\textbf{bsp\_sync}] \ldots \rangle$

Chosen solution: continuations (à la Blazy/Leroy)

$$s, \textbf{nprocs} \bullet \kappa \overset{i}{\rightharpoonup} s, \textbf{p} \bullet \kappa$$

$$s, \textbf{let } x = e_1 \textbf{ in } e_2 \bullet \kappa \overset{i}{\rightharpoonup} s, e_1 \bullet (\textbf{let } x = \_ \textbf{ in } e_2) \bullet \kappa$$

$$s, v \bullet (\textbf{let } x = \_ \textbf{ in } e_2) \bullet \kappa \overset{i}{\rightharpoonup} s[x \leftarrow v], e_2 \bullet \kappa$$

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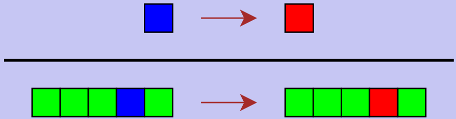
Local execution



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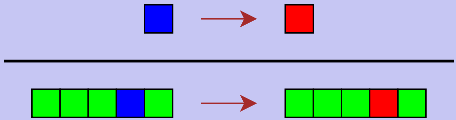
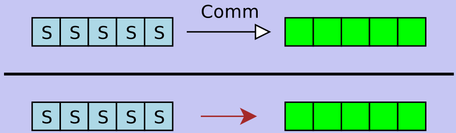
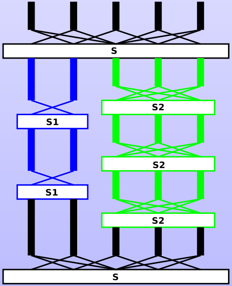


Synchronisation

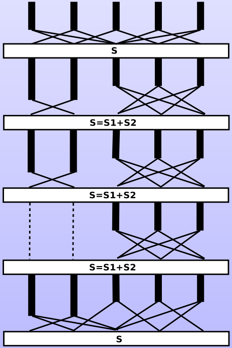


Big-Step Semantics with Subgroup Synchronisation

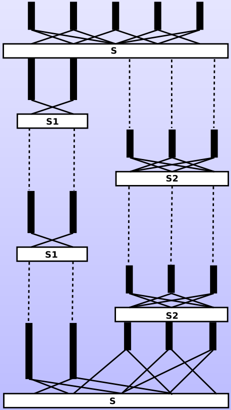
Machine execution



AllSub option

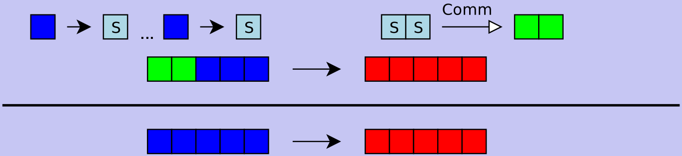


Diamond option



Big-Step Semantics with Subgroup Synchronisation

One synchronisation (**Diamond**)



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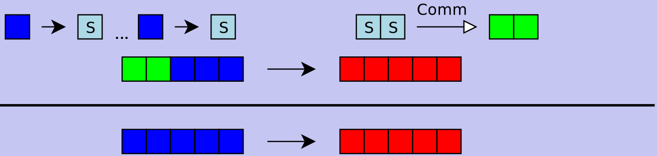
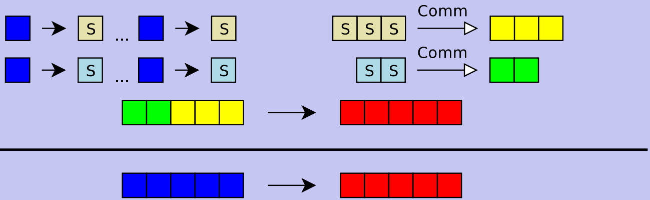


All synchronisations (**AllSub**)



Big-Step Semantics with Subgroup Synchronisation

Diamond variation:

$$\frac{\exists C\ \forall i \in C \quad s_i, e_i \Downarrow^i s'_i, \textbf{SYNC}(C, e'_i) \qquad \textbf{CommDia}\{C, (s'_0, e'_0), \ldots (s'_{\mathbf{p}-1}, e'_{\mathbf{p}-1})\} \Downarrow_{Diam} (s''_0, v_0), \ldots (s''_{\mathbf{p}-1}, v_{\mathbf{p}-1})}{(s_0, e_0), \ldots (s_{\mathbf{p}-1}, e_{\mathbf{p}-1}) \Downarrow_{Diam} (s''_0, v_0), \ldots (s''_{\mathbf{p}-1}, v_{\mathbf{p}-1})}$$

AllSub variation:

$$\frac{\{0, \ldots, \mathbf{p}-1\} = N \oplus C_1 \oplus \cdots \oplus C_k \qquad \forall i \in C_j \quad s_i, e_i \Downarrow^i s'_i, \textbf{SYNC}(C_j, e'_j) \qquad \forall i \in N \quad s_i, e_i \Downarrow^i s'_i, v \qquad \textbf{AllCommSub}\{C_1 \ldots C_k, v, (s'_0, e'_0), \ldots (s'_{\mathbf{p}-1}, e'_{\mathbf{p}-1})\} \Downarrow_{All} (s''_0, v_0), \ldots (s''_{\mathbf{p}-1}, v_{\mathbf{p}-1})}{(s_0, e_0), \ldots (s_{\mathbf{p}-1}, e_{\mathbf{p}-1}) \Downarrow_{All} (s''_0, v_0), \ldots (s''_{\mathbf{p}-1}, v_{\mathbf{p}-1})}$$

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$$\frac{s_i, e_i \bullet \kappa_i \xrightharpoonup{i} s'_i, e'_i \bullet \kappa'_i}{\langle (\ldots, (s_i, e_i \bullet \kappa_i), \ldots) \rangle \rightharpoonup \langle \ldots, (s'_i, e'_i \bullet \kappa'_i), \ldots \rangle}$$

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Results

Properties about semantics

- All semantics are confluent (Lemmas 1,2,7)
- Finite and diverging rules are mutually exclusive (4, 5)
- Finite Big-step and small-step semantics are equivalent (6)
- Two equivalent semantics for subgroup synchronisation (3)

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Benchmarks

Language	Rules	1	2	3	4	5	6	7	ALL
Our Why-ML	140	40	$\star$	$\star$	23	10	90	26	670
BSP-Why-ML	270	130	$\star$	$\star$	66	22	350	100	1300
With Subgroups	320	$\star$	416	531	85	35	490	446	1500
CompCert	513	1700	$\star$	$\star$	1200	100	?	1800	Big
IMP	30	12	$\star$	$\star$	14	8	53	11	135

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Analysis

1. Work(BSP-With-Subgroup) $\equiv$ 10 $\otimes$ Work(core-seq-language)
2. C language $\Rightarrow$ 10 years of work for team
3. Next talk $\Rightarrow$ 100 years (work for C+BSP+Subgroups)

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Outline

1. BSP Programming
2. Mechanised Semantics
3. Conclusion

Conclusion

BSP-WHY-ML

- BSP-WHY-ML is a BSP extension of WHY-ML
- BSP-WHY programs are transformed into WHY programs
- Subgroups take into account hierarchical architectures

Coq semantics

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- Proofs of equivalance, confluence, exclusive, etc.
- That do not scale ! (sizes of the proofs)

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Perspectives (Ongoing/Future Work)

Close to this subject

- Application to graph algorithms (Big-Data)
- Fully Mechanised BSP-WHY (diverging rules)
- Find how to better automate the proofs $\Rightarrow$ less human work
- Forget C, switch to a higher-level language such as Java

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Long term (team)

- Algorithms, semantics and verification tools for multi-ML
- Fully verified multi-BSP model checker ?
- Application to graph algorithms ?
- BSP/Skeleton abstract interpretation

Merci !