FraDeCo(P)P 2012

Proofs of pointer algorithms by inductive representation of graphs

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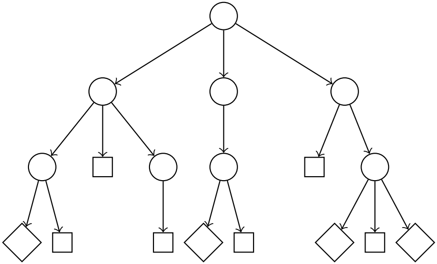
Outline

1. Context
   - Introduction
   - Approach
   - Verification of graph transformations
2. Schorr-Waite
   - Demo
   - Description
   - Verification
3. BDD
   - Description
   - Verification
4. Conclusion

First remark

Most usual data structures are tree-shaped with several kind of additional pointers:

- sharing pointers
- root pointers
- father pointers
- ...



For numerous algorithms, different views on data-structures can be used for the verification

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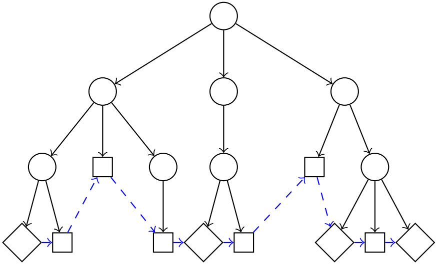

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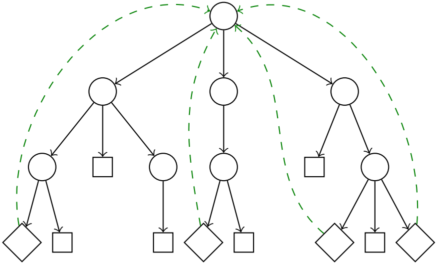

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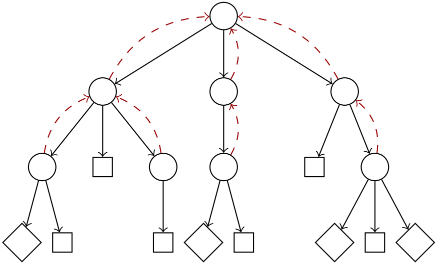

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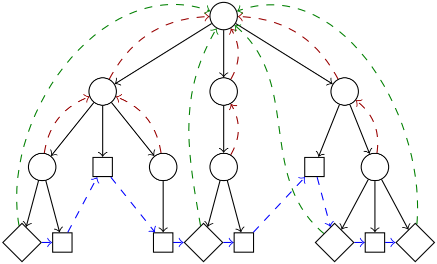

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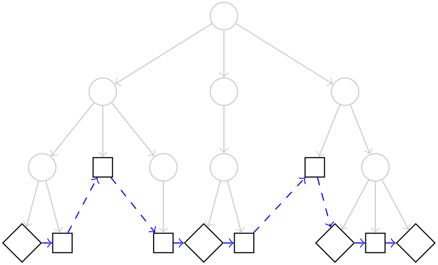

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Verification

Objective

Generate efficient and verified programs
(with pointers and/or mutable structures)

Safe code generation

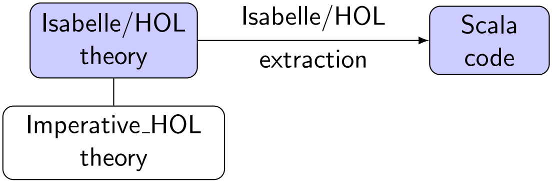
Abstract Language

- Arborescent data structure
  + pointers
- Properties verified by induction

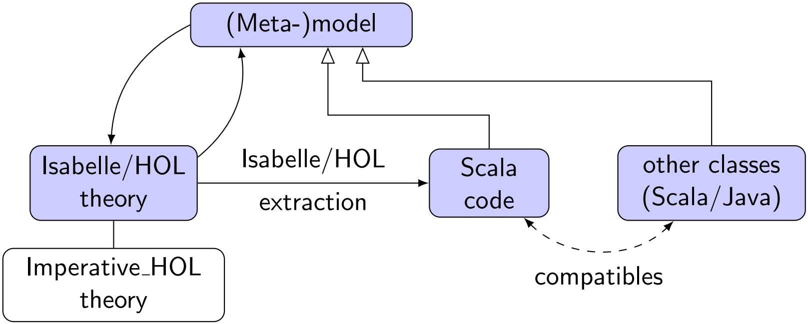
Imperative langage

- Efficient :
  - Sharing
  - Mutability
- Same properties

General frame



General frame



Related work

Graph representation

- Nodes and edges
- Coinduction
- Trees + pointers:
  - PALE [MS01]
  - Term-graph rewriting in Tom using relative positions [BB08]
  - Locally nameless encoding of lambda-terms [Cha09]
  - B+ trees functional representation with pointers [MM10]

Verification

- Proofs on raw heap and pointers
- Hoare (Separation) Logic [Rey02]

Case studies - Overview

Schorr-Waite

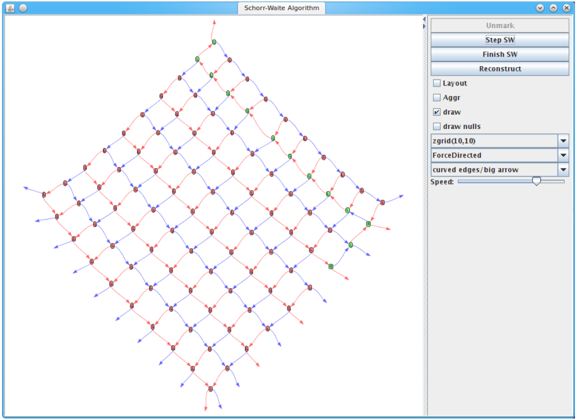
- Arbitrary rooted graph (with outgoing arity ≤ 2)
- High mutability
- Proof by simulation
- LOPSTR'2010 [GSMP10]

BDD

- Rooted acyclic graph (shared tree)
- References comparisons
- Direct proof
- TAAPSD'2010 [GS10], FoVeOOS'2011 [GS11]

Outline

Demo


Schorr-Waite Algorithm
Unmark
Step SW
Finish SW
Reconstruct
Layout
Aggr
draw
draw nulls
zgrid(10,10)
ForceDirected
curved edges/big arrow
Speed:

Algorithm features

Purpose

- Marking graphs without using more space (stack, ...)
- Traversing a tree by terminal recursivity and without stack

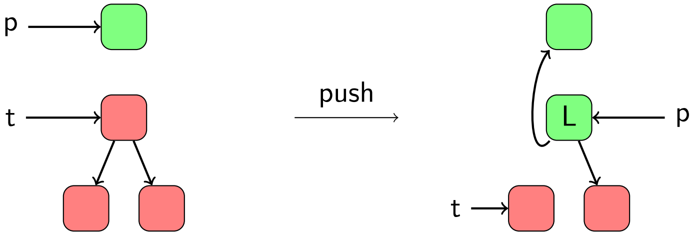
Use

Garbage collector, case study...

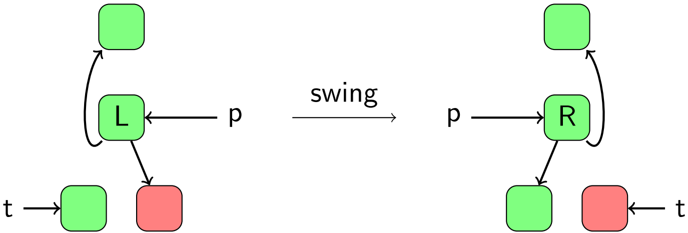
Principle

- Modification of the graph pointers to store the path to the root
- 2 variables containing the pointers :
  - t: to the current node
  - p: to the previously visited node

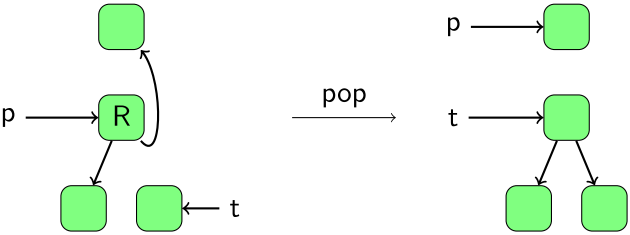
Steps : Push



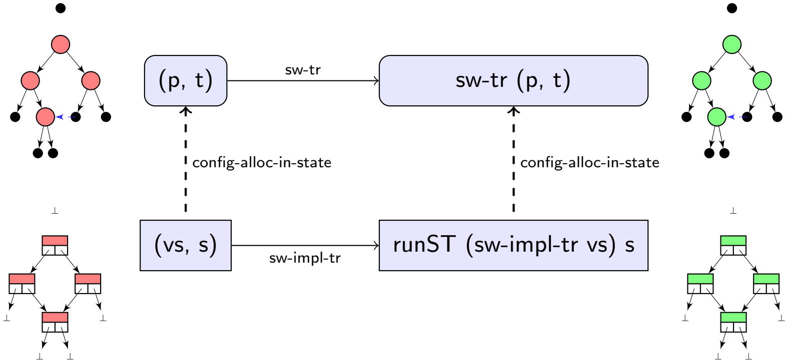
Steps : Swing



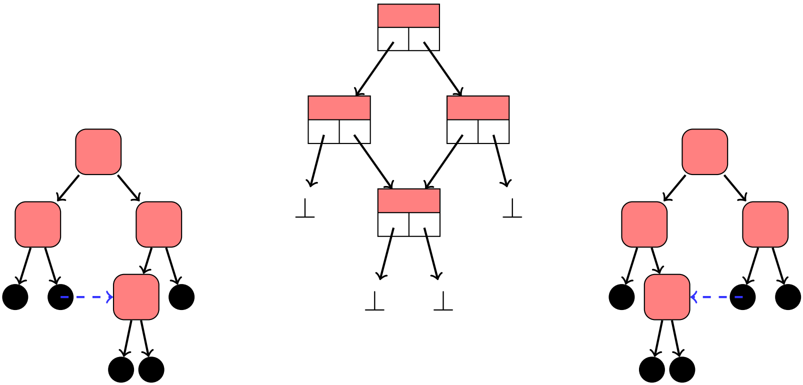
Steps : Pop



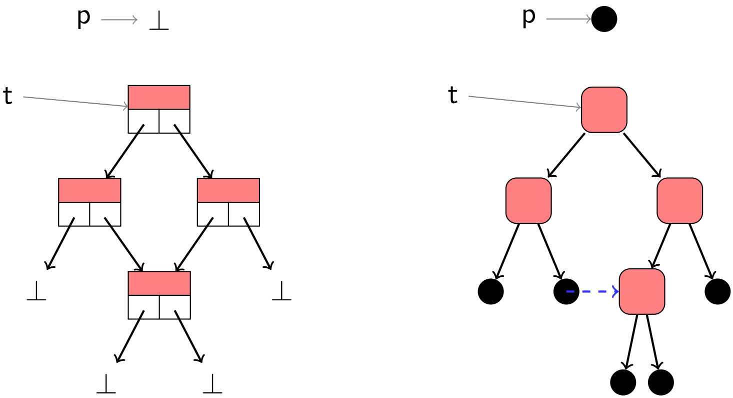
Proof by simulation



Choosing the spanning tree: Example with 2 possibility



The wrong one!



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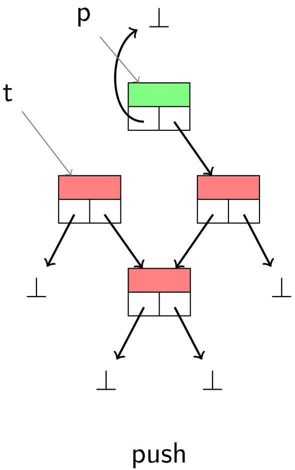

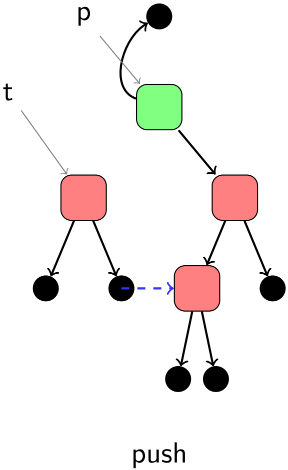

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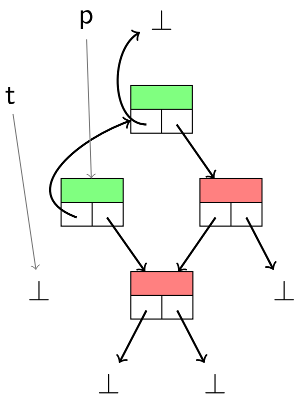
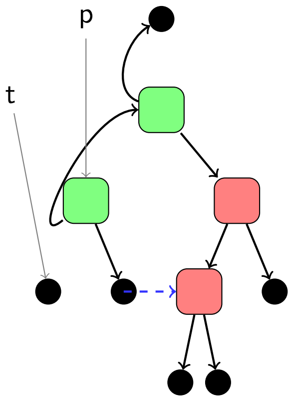


push



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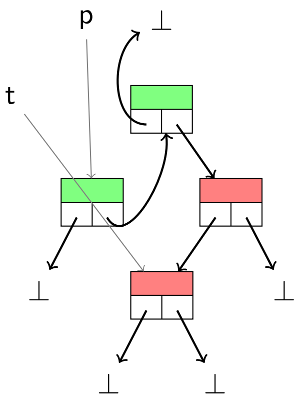
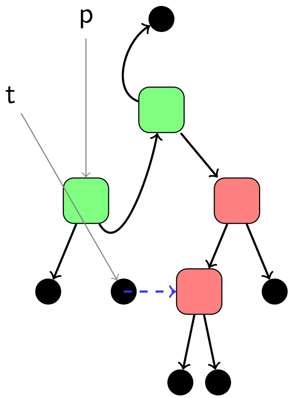


swing



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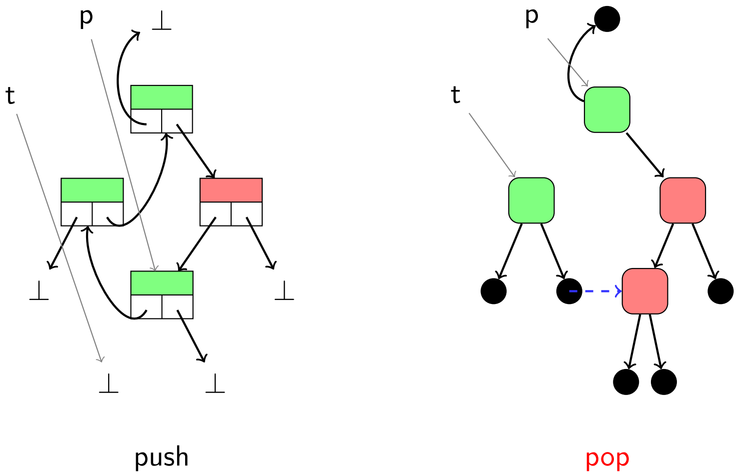

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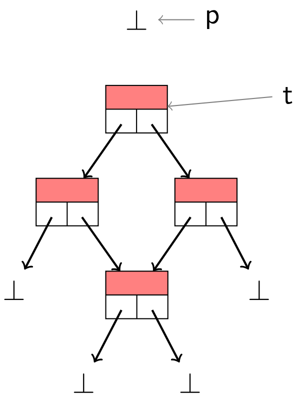

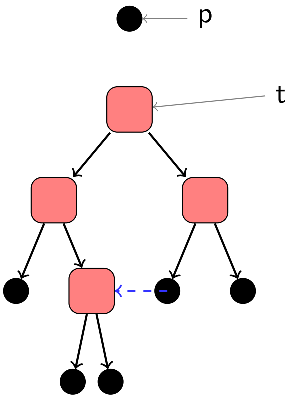

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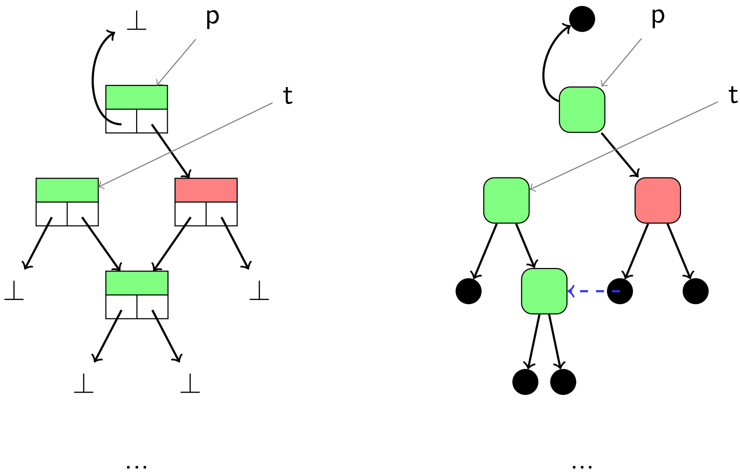

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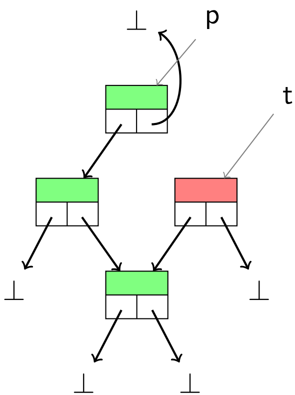
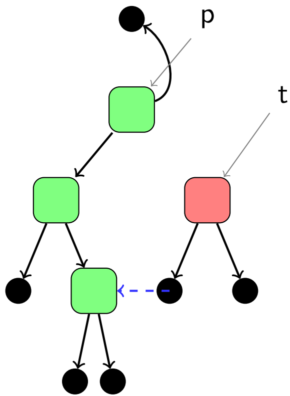


swing



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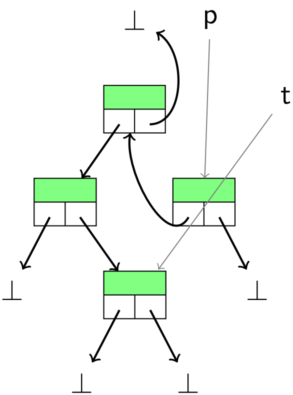

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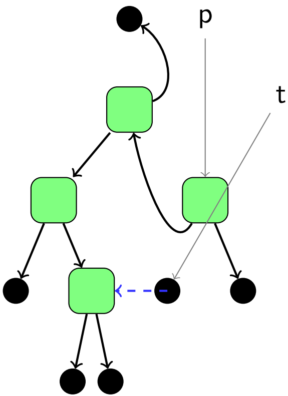

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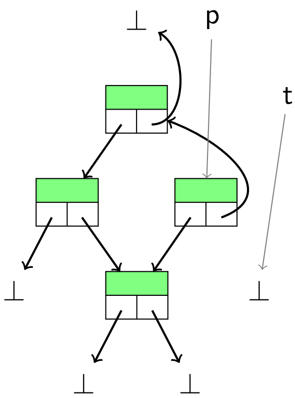

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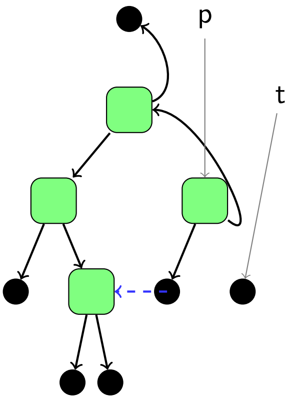
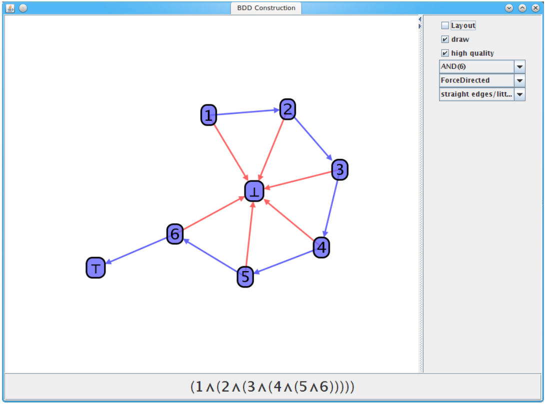


swing

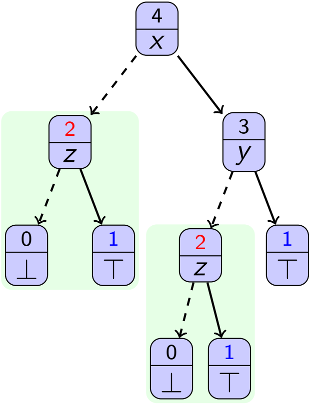
Outline

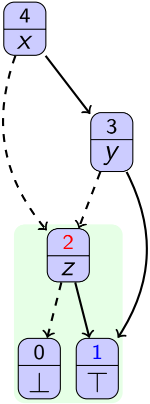
Demo


BDD Construction
Layout
draw
high quality
AND(6)
ForceDirected
straight edges/litt...
1
2
3
⊥
4
5
6
⊤
(1∧(2∧(3∧(4∧(5∧6)))))

Sharing

We add references to represent sharing. They also allow to add mutable content in the nodes.



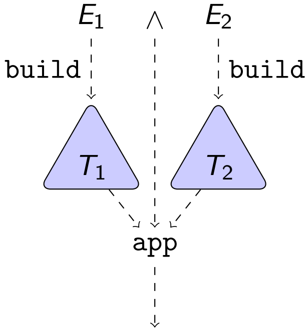


0	nbrefs $=$ 0 …
1	nbrefs $=$ 0 …
2	nbrefs $=$ 0 …
3	nbrefs $=$ 0 …
4	nbrefs $=$ 1 …
5	nbrefs $=$ - …

⋮

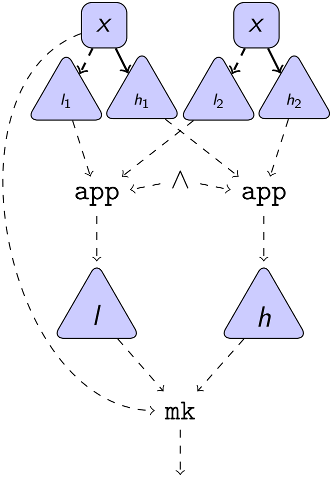
build

```
primrec build ::
    'v expr ⇒ (bool, 'v) rtree Heap
where
  build (Var i) = do{
   cf ← constLeaf False;
   ct ← constLeaf True;
   mk i cf ct
  }
| build (Const b) = (constLeaf b)
| build (BExpr bop e1 e2) = do{
   n1 ← build e1;
   n2 ← build e2;
   app bop (n1, n2)
  }
```



app

```
function app ::
  (bool ⇒ bool ⇒ bool)
  ⇒ ((bool, ′v) rtree ∗ (bool, ′v) rtree)
  ⇒ (bool, ′v) rtree Heap
where
  app bop (n1, n2) = do {
    if tpair is-leaf (n1, n2) then
      constLeaf (bop (leaf-val n1) (leaf-val n2))
    else do {
       let ((l1, h1), (l2, h2)) =
         select split-lh dup (n1, n2);
       l ← app bop (l1, l2);
       h ← app bop (h1, h2);
       mk (varOfLev (min-level (n1, n2))) l h
    }
  }
```



Memoization and Garbage Collection

Memoization

Records previous computations results to reuse them

- little change in functions and proofs

Garbage collection

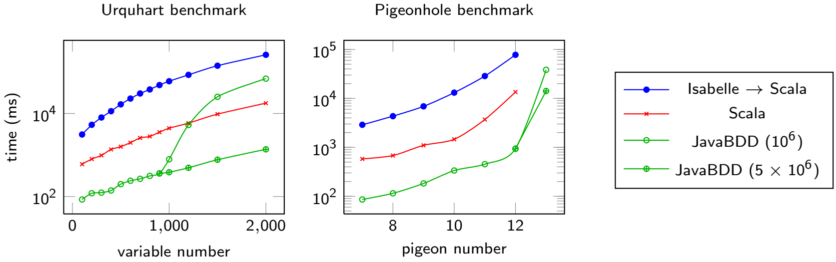
Removes no more used BDDs from the maximal sharing table

- several reference counter mutations in functions and proofs
- weakening of an invariant

Main theorem & benchmarks

Equivalent expressions construct the same BDD

lemma build-correct: $[\![\forall t \in \text{trees s1} \cup \text{trees s2. robdd-refs t};$
$\text{wf-heap s1}; \ \text{effect (build e1) s1 s1}' \text{ t1};$
$\text{wf-heap s2}; \ \text{effect (build e2) s2 s2}' \text{ t2}]\!]$
$\Longrightarrow \text{(interp-expr e1 = interp-expr e2)} \longleftrightarrow \text{struct-equal (t1, t2)}$



Outline

Conclusion

Approach

Representing graphs as (trees + pointers) to verify transformations

Schorr-Waite

- Arbitrary rooted graph (with outgoing arity ≤ 2)
- High mutability / Proof by simulation

BDD

- Rooted acyclic graph (shared tree)
- References comparisons / Direct proof

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Thanks for your attention

Emilie Balland and Paul Brauner.
Term-graph rewriting in Tom using relative positions.
In Ian Mackie, editor, *4th International Workshop on Computing with Terms and Graphs TERMGRAPH 2007 ENTCS*, volume 203 of *ENTCS*, pages 3–17, Braga Portugal, 2008. ELSEVIER.

Arthur Charguéraud.
The locally nameless representation.
Unpublished.
http://arthur.chargueraud.org/research/2009/ln/, July 2009.

Mathieu Giorgino and Martin Strecker.
Bdds verified in a proof assistant (preliminary report).
In A.V. Anisimov and M.S. Nikitchenko, editors, *Proc. TAAPSD*, Univ. Taras Shevchenko, Kiev, 2010.

Mathieu Giorgino and Martin Strecker.

Towards the verification of efficient bdd algorithms.
In *Formal Verification of Object-Oriented Software (FoVeOOS)*, 2011.

Mathieu Giorgino, Martin Strecker, Ralph Matthes, and Marc Pantel.
Verification of the Schorr-Waite algorithm – From trees to graphs.
In *Logic-Based Program Synthesis and Transformation (LOPSTR)*, 2010.
http://www.irit.fr/~Mathieu.Giorgino/Publications/GiSt2010SchorrWaite.html.

J. Gregory Malecha and Greg Morrisett.
Mechanized verification with sharing.
In *ICTAC*, pages 245–259, 2010.

Anders Møller and Michael I. Schwartzbach.
The pointer assertion logic engine.

In *Proc. ACM PLDI*, pages 221–231, 2001.

John C. Reynolds.
Separation logic: A logic for shared mutable data structures.
In *LICS '02: Proceedings of the 17th Annual IEEE Symposium on Logic in Computer Science*, pages 55–74, Washington, DC, USA, 2002. IEEE Computer Society.